Exercice series n°5 - Semester 1
2025 - 2026
Ring of Polynomials

Exercice 1:
Consider the following polynomial: $P(X) = 2X^4 - 9X^3 + 14X^2 - 9X + 2$

1. Justify whether P admits a zero root.
2. Show that for all $\lambda \in \mathbb{R}^* : \lambda$ is a root of $P \Rightarrow \frac{1}{\lambda}$ is a root of P.
3. Determine a root of P and deduce a factorization of P in $\mathbb{R}[X]$.

Exercice 2:

1. Let P a polynomial and a, b two distinct complex numbers. Compute in terms of $P(a)$ and of $P(b)$ the remainder of the division of P by $(X - a)(X - b)$.
2. Calculate the remainder of the Euclidean division of the polynomial $X^n + X + 1$ by the polynomial $(X - 1)^2$.
3. For which values of m the polynomial $P = (X + 1)^m - X^m - 1$ is divisible by the polynomial $Q = X^2 + X + 1$?

Exercice 3:
Let the polynomial

$$P(X) = X^6 - X^4 + 2X^3 - X + 1$$

Compute $P(1 + i)$ and $P(1 + \sqrt{3})$.

Exercice 4:

1. Determine the real or complex roots of $(X + 1)^6 - X^6$.
2. Let $a \in \mathbb{R}$ and let $P \in \mathbb{R}[X]$ defined by

$$P = (X + 1)^7 - X^7 - a$$

determine a so that P admit a multiple real root.

Exercice 5:
Let $P_1 \in \mathbb{R}$, $deg(P_1) \geq 1$. We set $P_2(X) = (P_1'(X))^2 - P_1(X)P_1''(X)$.

1. Let $\alpha \in \mathbb{R}$ a root of P_2.
Show that: if α is a root of P_1, then its multiplicity order is at least equal to 2 for both P_1 and P_2.
2. Let $\beta \in \mathbb{R}$ a root of P_1, with a multiplicity order equal to 2.
Show that β is a root of P_2, for which we have to determine the multiplicity order.
3. We choose $P_1(X) = X^2(2 + X^2)^{2n-1} - X^{4n}, n \in \mathbb{N}, n \geq 1$.
a. Determine all the complex roots of P_1, with indication of order.
b. Deduce a multiple root of the polynomial $P_2, n \geq 2$.
c. Determine the remainder of the Euclidean division by $(X^2 - 1)$ of the polynomial P_1.